

Careful calculation of thermodynamical functions of tachyon gas

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Abstract

We analyze several approaches to the thermodynamics of tachyon matter. The energy spectrum of tachyons $\varepsilon_k = \sqrt{k^2 - m^2}$ is defined at $k \geq m$ and it is not evident how to determine the tachyonic distribution function and calculate its thermodynamical parameters. Integrations within the range $k \in (m, \infty)$ yields no imaginary quantities and tachyonic thermodynamical functions at zero temperature satisfy the third law of thermodynamics. It is due to an anomalous term added to the pressure. This approach seems to be correct, however, exact analysis shows that the entropy may become negative at finite temperature. The only right choice is to perform integration within the range $k \in (0, \infty)$, taking extended distribution function $f_\varepsilon = 1$ and the energy spectrum $\varepsilon_k = 0$ when $k < m$. No imaginary quantity appears and the entropy reveals good behavior. The anomalous pressure of tachyons vanishes but this concept may play very important role in the thermodynamics of other forms of exotic matter.

1 Introduction

The concept of tachyon fields plays significant role in the modern research, where they often appear in the field theory, cosmology, theory of branes and strings with various applications [1, 2, 3, 4, 5]. Tachyons, are commonly

known as field instabilities whose energy spectrum is

$$\varepsilon_k = \sqrt{k^2 - m^2} \quad (1)$$

where m is the tachyon mass and we use the system of standard relativistic units $c_{light} = \hbar = k_B = 1$. Of course, it is highly desirable to consider such substances as an ideal gas of particles with a given energy spectrum because this model allows to calculate all thermodynamical parameters of exotic matter.

A system of many tachyons can be studied in the frames of statistical mechanics [6, 7], and thermodynamical functions of ideal tachyon Fermi and Bose gases are calculated [8, 9]. The properties of cold tachyon Fermi gas [10], its low-temperature behavior [11], tachyonic thermal excitations [12] and the hot tachyon gas [13] are also investigated.

Most peculiar behavior of tachyon gas concerns that fact that the system of tachyons may exist as a stable continuous medium when it satisfies the causality condition

$$c_s \leq 1 \quad (2)$$

The tachyon energy spectrum (1) is defined at $k \geq m$, so we have taken limits of integration in the range $m \leq k < \infty$ for all thermodynamical quantities [10]. The latter fact may result in contradiction to the third law of thermodynamics, and we check it in the present paper. If so, an anomalous pressure term will be necessary to avoid this trouble. However, appearance of the anomalous pressure implies that our previous analysis of cold tachyon Fermi gas [10] is incorrect. Either the anomalous pressure term is really present, or it is necessary to reformulate the theory and find right definitions for the energy spectrum and distribution function of tachyon gas.

2 Thermodynamical functions

Consider an ideal Fermi gas of N free particles enclosed in volume V . At finite temperature T its statistical sum is [14]

$$\ln Z = \pm \frac{\gamma}{2\pi^2} V \int_0^\infty \ln \left(1 \pm \exp \frac{\mu - \varepsilon_k}{T} \right) k^2 dk \quad (3)$$

where the upper and lower signs correspond to Fermi and Bose statistics, respectively.

The distribution function is

$$f_\varepsilon = \frac{1}{\exp[(\varepsilon_k - \mu)/T] \pm 1} \quad (4)$$

depends on the single-particle energy spectrum ε_k and chemical potential μ . According to standard formulas we determine the thermodynamical potential

$$\Omega = -T \ln Z \quad (5)$$

and the Helmholtz free energy

$$F = \Omega + \mu N \quad (6)$$

together with the particle number density

$$n = \frac{N}{V} = -\frac{1}{V} \left(\frac{\partial \Omega}{\partial \mu} \right)_{V,T} = \frac{1}{V} \frac{\partial (T \ln Z)_{V,T}}{\partial \mu} = \frac{\gamma}{2\pi^2} \int_0^\infty f_\varepsilon k^2 dk \quad (7)$$

the energy density

$$E = -\frac{T^2}{V} \frac{\partial (F/T)_{V,\mu}}{\partial T} = \frac{T^2}{V} \frac{\partial (\ln Z)_{V,\mu}}{\partial T} + \mu n = \frac{\gamma}{2\pi^2} \int_0^\infty f_\varepsilon \varepsilon_k k^2 dk \quad (8)$$

the entropy

$$S = V \left(\frac{\partial P}{\partial T} \right)_{V,\mu} = V \frac{\partial (T \ln Z)_{V,\mu}}{\partial T} = \frac{EV + P - \mu N}{T} \quad (9)$$

the specific heat

$$C_V = T \left(\frac{\partial S}{\partial T} \right)_V = \left(\frac{\partial E}{\partial T} \right)_V \quad (10)$$

and the pressure

$$P = -\frac{\Omega}{V} = \frac{T}{V} \ln Z = \pm \frac{\gamma}{2\pi^2} T \int_0^\infty \ln \left(1 \pm \exp \frac{\mu - \varepsilon_k}{T} \right) k^2 dk \quad (11)$$

Integrating (12) by parts, we have

$$P = \bar{P} + \tilde{P} \quad (12)$$

where

$$\bar{P} = \frac{\gamma}{6\pi^2} \int_0^\infty f_\varepsilon \frac{d\varepsilon_k}{dk} k^3 dk \quad (13)$$

and

$$\tilde{P} = \pm \frac{\gamma T}{6\pi^2} k^3 \ln \left(1 \pm \exp \frac{\mu - \varepsilon_k}{T} \right) \Big|_0^\infty \quad (14)$$

The latter anomalous pressure (14) is not reflected in the specific heat (10) but the entropy (9) will be changed. It is clear that the anomalous term (14) vanishes for ordinary subluminal particles (bradyons) with the energy spectrum

$$\varepsilon_k = \sqrt{k^2 + m^2} \quad (15)$$

and their pressure (12) is determined by the well-known formula

$$P = \bar{P} = \frac{\gamma}{6\pi^2} \int_0^\infty f_\varepsilon \frac{d\varepsilon_k}{dk} k^3 dk \quad (16)$$

According to the third law Nernst heat theorem [15], the entropy (9) must vanish at zero temperature.

$$S|_{T=0} = 0 \quad (17)$$

We should always bear in mind this fact when we analyze the statistical mechanics of tachyon ideal gas.

3 Pressure of cold tachyon Fermi gas

3.1 Appearance of imaginary terms

Consider a tachyon Fermi gas at zero temperature, when its distribution function (4) degenerates to the Heaviside step

$$f_\varepsilon = \Theta(\varepsilon_F - \varepsilon_k) \quad (18)$$

where

$$\mu|_{T=0} = \varepsilon_F = \sqrt{k_F^2 - m^2} \quad (19)$$

is the Fermi energy and k_F is the Fermi momentum. Substituting f_ε (18) in (7), (8) and (16) we find equation for the Fermi momentum

$$n = \frac{\gamma}{6\pi^2} k_F^3 \quad (20)$$

and the energy density and pressure

$$E = \frac{\gamma}{2\pi^2} \left(\frac{\varepsilon_F k_F^3}{4} - \frac{m^2 \varepsilon_F k_F}{8} - \frac{m^4}{8} \ln \frac{k_F + \varepsilon_F}{m} \right) + E_0 \quad (21)$$

$$P = \frac{\gamma}{2\pi^2} \left(\frac{\varepsilon_F k_F^3}{12} + \frac{m^2 \varepsilon_F k_F}{8} + \frac{m^4}{8} \ln \frac{k_F + \varepsilon_F}{m} \right) + P_0 \quad (22)$$

where

$$E_0 = -P_0 = i \frac{\gamma}{32\pi} m^4 \quad (23)$$

The imaginary term (23) is constant and it is not reflected in the sound speed

$$c_s^2 = \frac{dP}{dE} = \frac{dP}{dk_F} / \frac{dE}{dk_F} = \frac{1}{3} \frac{k_F^2}{k_F^2 - m^2} \quad (24)$$

The latter satisfies the causality condition (2) when

$$k_F \geq \sqrt{\frac{3}{2}} m \quad (25)$$

Moreover, substituting (19)-(23) in formula (9), we find that, in fact, the entropy of cold tachyon gas vanishes because

$$E + P - \varepsilon_F n = 0 \quad (26)$$

that agrees with the Nernst heat theorem (third law of thermodynamics) [15].

3.2 Elimination of imaginary terms by changing limits of integration

However, we do not know what physical meaning should pertain to imaginary part (23) of the energy density (21) and pressure (22). Thus, the above analysis is looking like no more than a mathematical trick. The tachyon

energy spectrum (1) is defined only at $k \geq m$, and it is reasonable to redefine limits of integration

$$\int_0^\infty \dots dk \rightarrow \int_m^\infty \dots dk \quad (27)$$

Substituting (27) in (7), (8) and (16) we find that the energy density and the pressure are real quantities

$$E = \frac{\gamma}{2\pi^2} \left(\frac{\varepsilon_F k_F^3}{4} - \frac{m^2 \varepsilon_F k_F}{8} - \frac{m^4}{8} \ln \frac{k_F + \varepsilon_F}{m} \right) \quad (28)$$

$$P = \frac{\gamma}{2\pi^2} \left(\frac{\varepsilon_F k_F^3}{12} + \frac{m^2 \varepsilon_F k_F}{8} + \frac{m^4}{8} \ln \frac{k_F + \varepsilon_F}{m} \right) \quad (29)$$

while the Fermi momentum is defined from equation

$$n = \frac{\gamma}{6\pi^2} (k_F^3 - m^3) \quad (30)$$

The sound speed is determined by the same formula (24), and the causality (2) is satisfied under the same condition (25). The only trouble is that according to (9) and (28)-(30), the entropy is finite because

$$TS = V(P + E - \varepsilon_F n) = \frac{\gamma V}{6\pi^2} \varepsilon_F m^3 \neq 0 \quad (31)$$

that contradicts to the third law of thermodynamics (17). The violation of this law in a tachyon system had been already emphasized [6]. Is it really so sad? Or the third law is still valid?

3.3 Inclusion of anomalous pressure

This problem can be resolved in the following way. Operating with limits of integration (27), we should not forget to check the anomalous pressure term (14) which has the form

$$\tilde{P} = \mp \frac{\gamma}{6\pi^2} T m^3 \ln \left[1 \pm \exp \left(\frac{\mu}{T} \right) \right] \leq 0 \quad (32)$$

It equals to zero when $k \in (0, \infty)$ but will be finite when $k \in (m, \infty)$. It is always non-positive for bosons and fermions, and for a tachyon Fermi gas at zero temperature it is estimated as

$$\tilde{P}_0 = \tilde{P}|_{T=0} = -\frac{\gamma}{6\pi^2} \varepsilon_F m^3 \quad (33)$$

Adding (33) to (29) we find the proper pressure of the cold tachyon Fermi gas

$$P = \frac{\gamma}{4\pi^2} \left(\frac{\varepsilon_F k_F^3}{6} + \frac{m^2 \varepsilon_F k_F}{4} + \frac{m^4}{4} \ln \frac{k_F + \varepsilon_F}{m} \right) - \frac{\gamma}{6\pi^2} \varepsilon_F m^3 \quad (34)$$

Taking the energy density (28), the particle number density (30) and the right-defined pressure (34), we see that condition (26) takes place, and, hence, the entropy (9) vanishes $S = 0$.

The anomalous term (33) added to the pressure (34) results in sufficient changes of the tachyon gas parameters. Now the pressure P never exceeds the energy density E , while the sound speed

$$c_s^2 = \frac{dP}{dE} = \frac{1}{3} \frac{k_F^2 + k_F m + m^2}{(k_F + m) k_F} \quad (35)$$

is always subluminal at all $k_F \geq m$. It may seem that the previous research of tachyon Fermi gas [10, 12] is wrong because we have not taken into account the evident anomalous pressure (33). However, we need to check whether this concept is working at finite temperature. Otherwise, we need to look for the right approach to the statistical mechanics of tachyons.

4 Tachyon pressure at finite temperature

Consider the tachyon anomalous pressure (32) at finite temperature. It is

$$\tilde{P} = -\frac{\gamma}{6\pi^2} m^3 \left[\mu + T \exp \left(\frac{-\mu}{T} \right) \right] \quad (36)$$

for a Fermi gas at low temperature $T \ll \mu$, while at zero temperature the pressure (36) is reduced to (33).

The anomalous pressure term (36) is incorporated in the entropy (9), namely

$$S = \frac{V}{T} \left(E - \mu n + \bar{P} + \tilde{P} \right) \quad (37)$$

At zero temperature we have $S|_{T=0}$ because $\mu|_{T=0} = \varepsilon_F$ and

$$E|_{T=0} - \varepsilon_F n + \bar{P}|_{T=0} + \tilde{P}|_{T=0} = 0 \quad (38)$$

according to (28), (30) and (34).

Subtracting (38) from (37) we have

$$S = \bar{S} + \frac{V}{T} (\tilde{P} - \tilde{P}_0) \quad (39)$$

where

$$\bar{S} = \frac{V}{T} (E - E|_{T=0} - \mu n + \varepsilon_F n + \bar{P} - \bar{P}|_{T=0}) \quad (40)$$

is the entropy of tachyon gas when the anomalous pressure (14) is not taken into account. It is clear that $\bar{S}|_{T=0} \neq 0$.

The chemical potential of tachyon Fermi at low temperature acquires a quadratic dependence on temperature [11]

$$\mu = \varepsilon_F \left(1 - \frac{\pi^2}{6} \frac{k_F^2 + \varepsilon_F^2}{k_F^2 \varepsilon_F^2} T^2 \right) \quad (41)$$

and the energy density and pressure are expanded in a power series of T :

$$E = E|_{T=0} + O(T^2) \quad \bar{P} = \bar{P}|_{T=0} + O(T^2) \quad (42)$$

It results in a linear dependence of the entropy density on temperature

$$\bar{S} = \frac{\gamma V}{6} \varepsilon_F k_F T \quad (43)$$

Appearance of anomalous pressure \tilde{P} results in additional contribution to the entropy (39). Substituting (41) in (36), we find a low temperature expansion of the anomalous pressure

$$\tilde{P} = \tilde{P}_0 + \frac{\gamma m^3}{6} \left[\frac{\varepsilon_F}{6} \frac{k_F^2 + \varepsilon_F^2}{k_F^2 \varepsilon_F^2} T - \frac{1}{\pi^2} \exp\left(\frac{-\varepsilon_F}{T}\right) \right] \quad (44)$$

Thus, substituting (43) and (44) in (39) we find the proper entropy

$$S = \frac{\gamma m^3 V}{6\pi^2} \left[\pi^2 \left(\frac{k_F}{m^3} + \frac{1}{6} \frac{k_F^2 + \varepsilon_F^2}{k_F^2 \varepsilon_F^2} \right) \varepsilon_F T - \exp\left(\frac{-\varepsilon_F}{T}\right) \right] \quad (45)$$

Consider function

$$g(T) = \lambda \frac{T}{\varepsilon_F} - \exp\left(\frac{-\varepsilon_F}{T}\right) \quad (46)$$

where

$$\lambda = \pi^2 \left(\frac{k_F}{m^3} + \frac{1}{6} \frac{k_F^2 + \varepsilon_F^2}{k_F^2 \varepsilon_F^2} \right) \varepsilon_F^2 \quad (47)$$

This function is negative $g(T) < 0$ when

$$\lambda < \frac{\varepsilon_F}{T} \exp \left(\frac{-\varepsilon_F}{T} \right) \quad (48)$$

where $\lambda \ll 1$ because $T \ll \varepsilon_F$. The dependence of critical ratio T/ε_F vs. λ is given in Fig. 1. Inequality (48) is realized when $\lambda < 1/e$. Parameter λ (47) can be arbitrary small when $k_F \rightarrow m$ and $\varepsilon_F \rightarrow 0$. Hence, function g (46) and the entropy S (45) will become negative at finite temperature when $k_F \rightarrow m$, although $S = g(T) = 0$ when $T \rightarrow 0$.

Now the definition of the entropy (45) contradicts to the laws of thermodynamics and it implies that the concept of anomalous pressure term (32) is not working here. The problem is hidden in our definition of the distribution function (4) and the limits of integration (27).

5 Discussion: right thermodynamical functions of tachyons

When we calculate the thermodynamical functions of a cold Fermi gas with the tachyon energy spectrum (1), the energy density (21) and pressure (22) may include imaginary parts. Imaginary terms does not appear if we change limits of integration (27), while an anomalous real term (32) will be added to pressure. This term is absolutely necessary here because the third law of thermodynamics must be satisfied (the entropy $S = 0$ at zero temperature $T = 0$). However, this term results in negative entropy of tachyon Fermi gas at finite temperature (45).

The only possible way to satisfy the third law of thermodynamics and to avoid imaginary quantities, is to perform integration within regular limits $k \in (0, \infty)$, however, taking the tachyonic energy spectrum in the form

$$\varepsilon_k = \begin{cases} \sqrt{k^2 - m^2} & k \geq m \\ 0 & k < m \end{cases} \quad (49)$$

and the distribution function in the form

$$f_\varepsilon = \begin{cases} 1 / \{ \exp [(\varepsilon_k - \mu) / T] \pm 1 \} & k \geq m \\ 1 & k < m \end{cases} \quad (50)$$

rather than

$$f_\varepsilon = \begin{cases} 1 / \{ \exp [(\varepsilon_k - \mu) / T] \pm 1 \} & k \geq m \\ 0 & k < m \end{cases} \quad (51)$$

Thus, substituting (53) in (7), we have

$$n = \frac{\gamma}{2\pi^2} \int_m^\infty f_\varepsilon k^2 dk + \frac{\gamma}{6\pi^2} m^3 \quad (52)$$

The distribution function (50) of a Fermi gas at zero temperature is reduced to

$$f_\varepsilon = \begin{cases} \Theta(\varepsilon_F - \varepsilon_k) & k \geq m \\ 1 & k < m \end{cases} \quad (53)$$

Substituting (49) and (53) in (8), (13) and (14), we find obtain the same energy density (28) and pressure (29), while the anomalous term (14) vanishes at all.

According to (52), the particle number density is determined by formula (20) rather than (30). As a result the entropy (9) vanishes because $E + P - \varepsilon_F n = 0$, and, contrary to the previous statement [6], the third law of thermodynamics is not violated.

Formula (20) implies that the Fermi momentum should be defined as

$$k_F = \left(\frac{\gamma n}{6\pi^2} \right)^{1/3} \geq m \quad (54)$$

rather than

$$k_F = \left(\frac{\gamma n}{6\pi^2} + m \right)^{1/3} \geq m \quad (55)$$

and that the lowest possible particle number density is

$$n_{\min} = \frac{\gamma}{6\pi^2} m^3 \quad (56)$$

rather than $n_{\min} = 0$. However, these changes concern the very link between the Fermi momentum k_F and the particle number density n , while it is not reflected in the energy density and pressure, which depend only on k_F without regard of how k_F is defined. Indeed, the sound speed is determined by the same formula (24), and the causality (2) is satisfied under the same condition

$k_F \geq \sqrt{3/2}m$ (25). The ratio P/E can exceed the unit, and all peculiar behavior of cold tachyon Fermi gas [10] remains valid.

The only consequence of formula (52) may concern the low-temperature expansion of the Fermi level [11]. Let us define dimensionless variables

$$x = \frac{\varepsilon_k}{T} \quad \lambda = \frac{\mu}{T} \quad \beta = \frac{m}{T} \quad (57)$$

and write formula (52) for a Fermi-Dirac distribution function at finite temperature:

$$n = \frac{\gamma T^3}{2\pi^2} \int_0^\infty \frac{\sqrt{x^2 + \beta^2} x dx}{\exp(x - \lambda) + 1} + \frac{\gamma}{6\pi^2} m^3 \quad (58)$$

An arbitrary integral

$$J(\lambda) = \int_0^\infty \frac{g(x) dx}{\exp(x - \lambda) + 1} \quad (59)$$

is expanded at low temperature ($\lambda \gg 1$) in the following series [11, 16]

$$J(\lambda) = G(\lambda) - G(0) + g'(\lambda) \frac{\pi^2}{6} + g'''(\lambda) \frac{7\pi^4}{360} + \dots \quad (60)$$

So we immediately calculate

$$\int_0^\infty \frac{\sqrt{x^2 + \beta^2} x dx}{\exp(x - \lambda) + 1} = \frac{1}{3} \sqrt{(\lambda^2 + \beta^2)^3} - \frac{1}{3} \beta^3 + \frac{\pi^2}{6} \frac{2\lambda^2 + \beta^2}{\sqrt{\lambda^2 + \beta^2}} \quad (61)$$

and, substituting (61) in (58), we obtain

$$n = \frac{\gamma}{6\pi^2} q^3 + \frac{\gamma}{12} \frac{2q^2 - m^2}{q} T^2 \quad (62)$$

where

$$q = \sqrt{\mu^2 + m^2} \quad \mu \xrightarrow{T \rightarrow 0} \sqrt{\varepsilon_F^2 + m^2} = k_F \quad (63)$$

Taking in to account that fact that the number of particles is conserved and that the particle number density at zero temperature is defined by formula (20), we find

$$q = k_F \left(1 - \frac{\pi^2}{6} \frac{k_F^2 + \varepsilon_F^2}{k_F^4} T^2 \right) \quad (64)$$

$$\mu = \varepsilon_F \left(1 - \frac{\pi^2}{6} \frac{k_F^2 + \varepsilon_F^2}{k_F^2 \varepsilon_F^2} T^2 \right) \quad (65)$$

It should be noted that the same expressions (64)-(65) are derived if we redefine the particle number density by formula (30) because the terms with m^3 are mutually annihilated in expansion (61)-(62). Formula (30) was used in our analysis of low-temperature expansion for the Fermi level of tachyon [11]. Thus, all previous results are valid, and the entropy and specific heat of tachyon Fermi gas are determined by formula [11]

$$C_V = S = \frac{\gamma V}{6} \varepsilon_F k_F T \quad (66)$$

The energy density, pressure, entropy and specific heat of tachyonic excitations [12] do remain right defined. The only correction concerns the particle number density, which is now

$$n = \frac{\gamma T^3}{2\pi^2} \int_0^\infty \frac{\sqrt{x^2 + \beta^2} x dx}{\exp x + 1} + n_0 = \frac{\gamma T^3}{2\pi^2} \left(\int_0^\infty \frac{\sqrt{x^2 + \beta^2} x dx}{\exp x + 1} + \frac{\beta^3}{3} \right) \quad (67)$$

where

$$n_0 = \frac{\gamma}{6\pi^2} m^3 \quad (68)$$

Its graph is shown in Fig. 2. However, there is no qualitative difference from the previous result (Fig. 1 in Ref. [12]).

As for the hot tachyon gas [13], its particle number density is now determined so

$$n = \frac{\gamma T^3}{2\pi^2} \exp\left(-\frac{\mu}{T}\right) \int_0^\infty \sqrt{x^2 + \beta^2} \exp(-x) x dx + n_0 \quad (69)$$

and the pressure and energy density are determined so

$$P = (n - n_0) T \quad (70)$$

$$E = (n - n_0) T J(T) \quad (71)$$

This problem needs special analysis.

We have introduced the concept of anomalous pressure (14). This pressure vanishes in a tachyon gas. However, the anomalous term (14) will be finite if the single-particle energy spectrum does not satisfy condition

$$\varepsilon_k \xrightarrow[k \rightarrow \infty]{} \infty \quad (72)$$

An example of such energy spectrum is [17]

$$\varepsilon_k \sim \frac{1}{k^\alpha} \quad \alpha > 0 \quad (73)$$

The concept of anomalous pressure (14) should be considered in detail in application to the exotic matter where it may play very significant role.

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Figure 1: Solution of inequality (48: T/ε_F vs. λ .

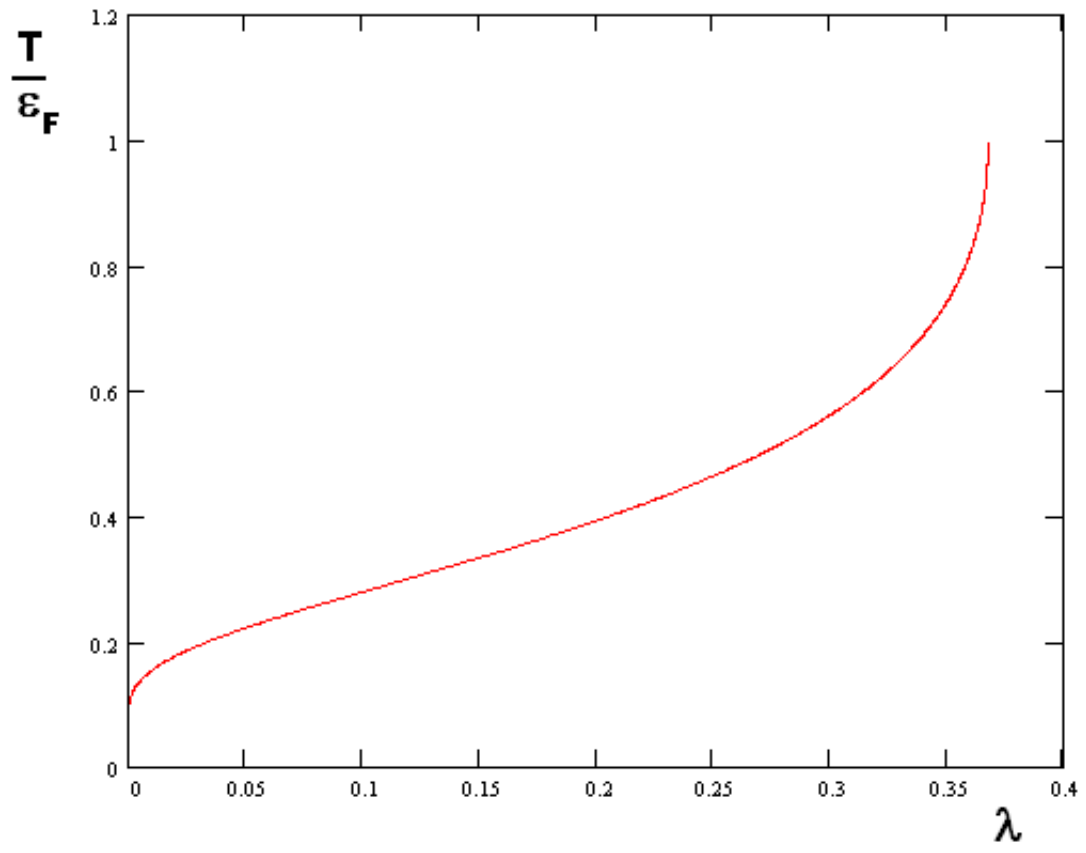


Figure 2: The particle number density n of ideal gases of tachyonic thermal excitations vs temperature variable $\beta = m/T$. Solid line: calculation according to formula (67, dashed line – according to Ref. [12].

